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**WORKING PAPER**

**LUMP-SUM TAXES IN A R&D MODEL**



# Lump-Sum Taxes in a R&D Model \*

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## Abstract

Is it possible to increase growth and welfare by raising taxes and disposing of the tax revenues? We show this may indeed be the case in a simple model with endogenous technical change, represented by an increase in the variety of intermediate goods.

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# 1 Introduction

The prediction that permanent variations in tax rates would give rise to different steady-state growth rates has long been an hallmark of the endogenous growth literature. In contrast to the older neoclassical framework, where long-run growth was exogenously determined by the rate of technical progress, these models predict that increases in tax rates would induce lower growth rates (see, for example the survey in Myles 2000 and Jones and Manuelli 2005). In particular many studies focus on R&D activities, a major driving force for growth, and to fiscal incentives for these activities, which are subsidized in many industrial countries. One limitation of these studies is that they often treat labor supply as inelastic, thereby abstracting from the decision to allocate time between work and leisure. In this paper we show a non obvious result for fiscal policy that is made possible by allowing for flexible labour supply in R&D models: the fact that lump taxes can have positive effects on growth even when the revenue is not used in a productive way.

It should be certainly possible to return the revenue to agents in such a way as to increase their welfare. However assuming as we do that the revenue is not returned allows us a closer view of the effect of fiscal policy. In particular it is often found in theoretical models that growth can be increased by subsidies to R&D financed through lump-sum taxes (see for example Barro and Sala-i-Martin (2004), chap. 6, or Zeng and Zhang (2002)). Here we show that with a flexible labour supply, lump-sum taxes can in themselves increase growth and welfare i.e. have a direct effect on them.

The mechanism which is at work is the following: a lump-sum tax induces a negative income effect thereby inducing agents to work more. More employment raises the returns to the R&D activity. Growth is therefore increased.

We conduct our study by using a standard model of endogenous technological progress with an infinitely lived representative agent, originally proposed by Romer (1990) and presented in Barro and Sala-i-Martin (2004), chap. 6. Entrepreneurs spend a fixed cost in order to develop new intermediate goods. Each chooses to produce the same amount of each intermediate good. Output in the final goods production sector is linear in the number of intermediate goods used so unbounded growth is possible. The basic difference in assumptions with respect to this benchmark model is that the decision to supply labour is explicitly analysed. We study the long-run ef-

fects effects of a lump-sum tax whose proceeds are thrown away and find that such a tax will increase growth and will increase welfare for a broad region of the parameters space. The intuition is simply that in our model, lump-sum taxes have an impact on the allocation of resources, because they influence labour supply and consequently the rate of return on capital and the rate of growth. In the example which we consider, the income effect of a wasted lump-sum tax causes households to consume less leisure and supply more labour. This causes an increase in the interest rate and the long-run rate of growth.

Our result is an example of second-best theory. The idea that taxes whose revenue is not used productively must reduce welfare is based on the first-best intuition that a waste of resources has a positive social cost. However the withdrawal of resources from productive use may have a social benefit in an economy in which there is imperfect competition, i.e. in a second-best environment.

Another contribution of our analysis is the following: as said above it is very frequent in works studying the effects of fiscal expenditures to assume financing by lump-sum taxation, taken to be non distortional, or to assume that proceeds of taxes are returned lump-sum (e.g. Devereux and Love (1995), Lin and Russo (1999), Turnovsky (2000), Zeng and Zhang (2002), or Haruyama and Itaya (2006)). However we show that, with elastic labour supply, through general equilibrium effects a lump-sum tax will change relative prices and therefore be indirectly distortional. In other words the effect on growth of a tax whose revenue is returned lump-sum will be different from the effect on growth of a tax whose proceeds are just thrown away and should therefore be studied separately.

The rest of the paper is organized as follows: in section 2 the model is presented, section 3 describes the equilibrium conditions which have to hold in the model, section 4 focuses on the balanced growth path characteristics of the model, section 5 analyses the effects of a lump-sum tax in the model, section 6 does some simple numerical calculations to show that such a tax can increase welfare for widely accepted estimates of the relevant parameters.

## 2 A Model

We present a simple growth model with endogenous technological progress consisting in the expansion of the variety of intermediate goods. Our mod-

eling starts with the version of Romer (1990) proposed by Barro and Sala-i-Martin (2004) and extends it by allowing for elastic labour supply.

## 2.1 Production

Following Spence (1976) and Dixit-Stiglitz (1977) the production function of firm  $i$  in the final good sector is given by :

$$Y(i) = AL(i)^{1-\alpha} \int_0^N x(i, j)^\alpha di \quad (1)$$

where  $Y(i)$  is the amount produced and  $L(i)$  is labour used by firm  $i$  and  $x(i, j)$  is the quantity this firm uses of the intermediate good indexed by  $j$ . For tractability both  $i$  and  $j$  are treated as continuous variables. We assume  $0 < \alpha < 1$ . The final good production sector is competitive and we assume a continuum of length one of identical firms. We can then suppress the index  $i$  to avoid notational clutter. Firms maximize profits given by

$$Y - wL - \int_0^N P(j)x(j)dj \quad (2)$$

where  $W$  is the wage and  $P(j)$  is the price of the intermediate good  $j$ . By profit maximization we have:

$$x(j) = L \left( \frac{A\alpha}{P(j)} \right)^{\frac{1}{1-\alpha}} \quad (3)$$

and

$$W = (1 - \alpha) \frac{Y}{L} \quad (4)$$

Since the firms in the final good production sector are competitive, their profits are zero in equilibrium. In contrast for the firms producing intermediate goods patent, the new intermediate good invented then earn monopoly profits for ever. The cost of production of the intermediate good  $j$ , once it has been invented, is given by one unit of final good. The value of the patent for the  $j$ th intermediate good  $v(j, t)$  at time  $t$  is the present discounted value of such profits. The value of the  $j$ th patent at time  $t$  is then

$$v(j, t) = \int_t^\infty (P(j) - 1)x(j)e^{-\bar{r}(s,t)(s-t)} ds \quad (5)$$

where  $\bar{r}(s, t)$  is the average interest rate during the period of time from  $t$  to  $s$ .

The inventor of the  $j$ th intermediate good chooses  $P(j)$  to maximize profits  $(P(j) - 1)x(j)$  where  $x(j)$  is given by 3, so for each  $j$ :

$$P(j) = P = \frac{1}{\alpha} \quad (6)$$

and

$$x(j) = x = LA^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} \quad (7)$$

so if labour supply is constant, and the interest rate is constant, we have substituting 6 and 7 in 5:

$$v(j, t) = LA^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} \left( \frac{1-\alpha}{\alpha} \right) \frac{1}{r} \quad (8)$$

We show below that labour supply and the interest rate are indeed constant in the balanced growth equilibrium.

The cost of development of new products is  $\eta$  and there is free entry of inventors. So by equating the value and the cost of inventions, in steady state we will have:

$$r = \frac{L}{\eta} A^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} \left( \frac{1-\alpha}{\alpha} \right) = \frac{L}{\eta} A^{\frac{1}{1-\alpha}} \alpha^{\frac{1+\alpha}{1-\alpha}} (1-\alpha) \quad (9)$$

Plugging equation 7 in equation 1 gives equation

$$Y = NLA^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} \quad (10)$$

and plugging 10 in 4 we have:

$$W = N(1-\alpha)A^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} \quad (11)$$

Thus the interest rate is proportional to labour supply and the wage does not depend on labour supply. With time the number of intermediate goods increases. The wage grows proportionally to the number of intermediate goods while the interest rate remains the same. Notice a higher labour supply implies a higher quantity of each intermediate goods in equilibrium. Inventors have larger sales and incomes and are therefore able and willing to pay a higher equilibrium interest rate on their debt  $\eta$ .

## 2.2 Households

We assume that consumers (again a continuum of length one) have utility  $U$  given by:

$$U = \int_{t=0}^{\infty} e^{-\rho t} (\ln(C) + h(L)) dt \quad (12)$$

Labour supply  $L$  ranges from zero to 1 and

$$h'(L) < 0, h''(L) < 0 \quad (13)$$

Since the utility function is additively separable in labour and consumption both leisure and consumption are normal goods.

Equation 14 gives the instantaneous budget constraint consumers face:

$$I = rF + WL - C - \tau_a \bar{F} \quad (14)$$

where  $I$  is investment and  $F$  is wealth. Households derive their income by loaning entrepreneurs their wealth (of which all have the same initial endowment) and by supplying labour  $L$  to firms in the final goods production sector, taking the interest rate  $r$  and the wage  $W$  as given. There are lump-sum taxes proportional to average wealth,  $\bar{F}$ , where given our normalization,  $F = \bar{F}$ . Agents, being atomistic, take these averages as variables beyond their control. In this sense these are lump-sum taxes.

The shadow value of wealth is  $\lambda = \frac{1}{C}$ . Optimization implies

$$-\frac{\dot{C}}{C} = \frac{\dot{\lambda}}{\lambda} = \rho - r \quad (15)$$

and

$$\lambda W + h'(L) = 0 \quad (16)$$

so

$$C = -\frac{W}{h'(L)} \quad (17)$$

We also have the transversality condition:

$$\lim_{t \rightarrow \infty} \lambda F \exp(-\rho t) = 0 \quad (18)$$

### 3 Market Equilibrium

The social budget constraint is

$$Y - xN = C + I + G \quad (19)$$

where  $G$  is public expenditure and total intermediate goods used  $xN$  is subtracted from final production  $Y$  to obtain total value added. All investment in the model is investment in research and development of new intermediate goods so  $\eta\dot{N} = I$  and  $F = \eta N$ . Thus, the social budget constraint is

$$\eta\gamma = \eta\dot{N} = I = Y - xN - C - G \quad (20)$$

We have assumed that government consumption  $G$  does not enter the utility function of consumers or the production function of firms. That is we want to study the effect of taxes, without considering the possible productive uses of the tax revenue. In this sense  $G$  is waste. This is a common assumption in macromodels. We rule out a market for government bonds and assume that the government runs a balanced budget. The flow government budget constraint can be written

$$G = \tau_a \eta \bar{N} \quad (21)$$

so the social budget constraint becomes

$$\eta\dot{N} = Y - xN - C - \tau_a \eta N \quad (22)$$

### 4 Balanced Growth

We consider only the balanced growth path (hence BGP) of the model labour supply and therefore the rate of interest (by 9) and the quantity produced of each intermediate good (by 7) are constant while the rate of growth of other variables is constant as well. When labour supply is constant, 11 implies that the wage and the number of intermediate goods grow at the same rate and 17 that consumption and the wage must grow at the same rate so we have:

$$\frac{\dot{C}}{C} = \frac{\dot{N}}{N} \equiv \gamma \quad (23)$$



where  $\gamma$  is defined as the BGP rate of growth. Equations 22, 15 and 23 imply that along a BGP:

$$\frac{\dot{C}}{C} = r - \rho = \frac{1}{\eta N} (Y - xN - C - \tau_a \eta N) = \frac{\dot{N}}{N} \quad (24)$$

using the factor exhaustion condition that the wage bill plus total interest payments is equal to GNP, that is  $Y - xN = WL + r\eta N$ , and substituting for  $C$  using equation 17 we can then write this BGP condition as

$$\frac{\dot{C}}{C} = r - \rho = r + \frac{1}{\eta N} \left( WL + \frac{W}{h'(L)} - \tau_a \eta N \right) = \frac{\dot{N}}{N} \quad (25)$$

The equation on the left-hand side of 25 is the Euler equation and gives us the BGP rate of growth of consumption as a function of labour only, as the interest rate is a positive linear function of labour (see 9). The equation on the right-hand side, i.e the social budget constraint, gives us the BGP rate of growth of intermediate goods again as a function of labour, given that  $W/N$  is a constant (see 11). We can then represent 25 as in figure 1, where the Euler equation is the curve labelled  $\frac{\dot{C}}{C}$ , while the social budget constraint is the graph labelled  $\frac{\dot{N}}{N}$ . Both curves are increasing in  $L$  but the  $\frac{\dot{N}}{N}$  curve is steeper than the  $\frac{\dot{C}}{C}$  curve. In fact the interest rate appears in the same fashion in both curves but there are two additional terms in  $L$ ,  $\frac{WL}{\eta N}$  and  $\frac{W}{\eta N h'(L)}$ , appearing only in the  $\frac{\dot{N}}{N}$  curve. These terms are both increasing in  $L$ , given 11 and 13. The fact that the  $\frac{\dot{N}}{N}$  curve is always steeper than the  $\frac{\dot{C}}{C}$  means that there is at most one point of intersection of the two curves. If the curves do intersect for  $0 < L < 1$  and the transversality condition is respected when they intersect, the point of intersection gives us the BGP rate of growth  $\gamma$  on the vertical axis and the BGP labour supply  $\tilde{L}$  on the horizontal axis. We conclude that if a BGP equilibrium exists it is unique. Notice that the transversality condition will always hold in a steady state as the amount of wealth and its shadow value grow at opposite rates. Below we introduce a specific version of  $h$  which implies existence of a solution  $\tilde{L}$  to 25.

## 5 Effects of Taxes

It is relatively simply to calculate the effects of taxes on labour supply in this model, because the wage is not affected by labour supply. Notice 25 can be simplified as

$$\frac{1}{\eta N} \left( WL + \frac{W}{h'(L)} - \tau_a \eta N \right) + \rho = 0 \quad (26)$$

Taking the total derivative of 26 with respect to  $\tau_a$  we obtain:

$$\frac{dL}{d\tau_a} = \frac{\eta N}{W \left( 1 - \frac{h''(L)}{(h'(L))^2} \right)} \quad (27)$$

Since  $h''(L) < 0$  the effect of the lump-sum tax on labour supply is positive. This can be interpreted as a simple income effect: for fixed labour supply, the tax would make households poorer so since both consumption and leisure are normal goods they consume less and offer more labour. It is easy to calculate the effect of the lump-sum tax on the rate of growth, since the rate of growth of consumption increases one for one with the interest rate and the interest rate is proportional to labour supply. Thus by 27, 9 and 11 if growth is balanced

$$\frac{d\gamma}{d\tau_a} = \frac{\alpha}{\left( 1 - \frac{h''(L)}{(h'(L))^2} \right)} \quad (28)$$

The effect of the tax on labour supply and growth is illustrated in Figure 2. An increase in the tax leaves the  $\frac{\dot{C}}{C}$  curve unaltered but shifts the  $\frac{\dot{N}}{N}$  curve down. The intersection point then moves north-east, that is both the BGP labour supply and the BGP rate of growth increase.

### 5.1 A Specific Example

We consider here a specific class of functions for the disutility of labour

$$h(L) = -\frac{L^{1+\chi}}{1+\chi}. \quad (29)$$

If  $\chi > 0$  labour is unpleasant and the marginal disutility of labour increases with labour. The balanced growth equilibrium condition 26 becomes:

$$\frac{1}{\eta N} \left( WL - \frac{W}{L^\chi} - \tau_a \eta N \right) + \rho = 0. \quad (30)$$

The left-hand side of the balanced growth equilibrium condition 30 is monotonically increasing in labour. In particular the expression increases monotonically from  $-\infty$  to  $\rho - \tau_a$  when  $L$  goes from 0 to 1. All of the terms in 30 are continuous in  $L$ , therefore a solution  $\tilde{L}$  exists provided  $\rho - \tau_a > 0$ . As seen above the solution of 30 is unique and there is a unique balanced growth equilibrium with  $L = \tilde{L}$ .

## 6 Welfare Analysis

Given  $\gamma$  the constant rate of growth and  $\tilde{L}$  it is possible to calculate lifetime utility  $V$  along a balanced growth path:

$$\begin{aligned} V &= \int_{t=0}^{\infty} e^{-\rho t} \left( \ln(C) + h(\tilde{L}) \right) dt = \int_{t=0}^{\infty} e^{-\rho t} \left( \ln(C(0) + \gamma t + h(\tilde{L})) \right) dt \\ &= \frac{1}{\rho} \left( \ln(C(0) + h(\tilde{L})) + \frac{\gamma}{\rho} \right) \end{aligned} \quad (31)$$

Where  $C(0)$  is consumption at time 0. Equations 31, 11 and 17 imply

$$V = \ln \left( N(0)(1 - \alpha) A^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} \right) - \ln(-h'(\tilde{L})) + h(\tilde{L}) + \frac{\gamma}{\rho} \quad (32)$$

where  $N(0)$  is the initial stock of patents. Thus the derivative of welfare with respect to  $\tau_a$  is

$$\frac{dV}{d\tau_a} = \left( \frac{h''(\tilde{L})}{-h'(\tilde{L})} + h'(\tilde{L}) + \frac{1}{\rho} \frac{d\gamma}{dL} \right) \frac{dL}{d\tau_a} \quad (33)$$

Note that  $\frac{dL}{d\tau_a} > 0$ , and that the effect of labour supply on the rate of growth

$$\frac{d\gamma}{dL} = \frac{dr}{dL} = \frac{r}{L} = \frac{1}{\eta} A^{\frac{1}{1-\alpha}} \alpha^{\frac{1+\alpha}{1-\alpha}} (1 - \alpha) > 0 \quad (34)$$

is positive and does not depend on  $L$ ,  $\rho$  or  $\tau_a$ . Thus the sign of the effect of  $\tau_a$  on welfare depends on  $\rho$  and the magnitude of the first two terms in large parentheses in 33 which are clearly negative. Consider again the specific function for the disutility of labour 29. In this case 33 becomes

$$\frac{dV}{d\tau_a} = \left( \frac{-\chi}{\tilde{L}} - \tilde{L}^\chi + \frac{1}{\rho} \frac{r}{\tilde{L}} \right) \frac{dL}{d\tau_a} \quad (35)$$

Equation 34 shows that the term  $\frac{r}{\tilde{L}}$  is the same positive constant for any  $L$ ,  $\rho$  or  $\tau_a$ . Inspection of the balanced growth equilibrium condition 30 shows that if  $\tau_a = 0$ ,  $\tilde{L}$  goes to 1 as  $\rho$  goes to 0. Thus as  $\rho$  goes to 0, the sum of the first two terms in large parentheses in 35 goes to  $-(1 + \chi)$  while the third term goes to  $+\infty$ . Therefore, a small wasted lump-sum tax causes increased welfare if  $\rho$  is low enough. An intuitive explanation of this result is that the wasted lump-sum tax causes increased growth which is very valuable if consumers are patient.

## 7 Is Waste Good For Reasonable Parameter Values?

The model analyzed above is clearly very simple. Further many modelling choices were made to ensure tractability. As such, the analysis can be considered a proof that rational expectations and the absence of spillovers does not logically imply that wasteful government spending is a bad thing. We now show that this happens for ranges of parameter values consistent with estimates available in the literature. We attempt to pin down the parameters in equation 35 with data. First we replace the unobservable  $\rho$  with  $r - \gamma$ . Reasonable estimates of  $\gamma$  range from 2% to 3% per year (the average post World War II rate of growth of per capita GNP in the USA and in Europe). Reasonable estimates of  $r$  are controversial, but many economists think that 5% per year is reasonable. Thus generally accepted estimates of  $\frac{r}{\rho}$  are on the order of  $\frac{5}{3}$  to 2.5. Estimates of  $\tilde{L}$  depend entirely on the interpretation of the maximum possible labour supply, 1 in the model. If it is interpreted as 24 hours a day 7 days a week and so 8760 hours a year, reasonable estimates of  $\tilde{L}$  range from  $\frac{1}{4.38}$  for the US to lower in Europe.  $\chi$  can in principle be estimated

by estimating the compensated elasticity of labour supply.<sup>1</sup> Estimates of the compensated elasticity of labour supply range from roughly zero to roughly 0.5 for men and from 0.5 to 1.65 for women (Pencavel 1986, Killingsworth and Heckman 1986). Clearly, if labour supply is inelastic, standard results obtain. For  $\gamma = 3\%$ ,  $\tilde{L} = \frac{1}{4.38}$ , and  $\chi = 2$ , the term in parentheses in equation becomes  $2.14 > 0$ . In conclusion our analysis suggests that waste could increase welfare for reasonable parameter values.

## 8 Conclusions

We offer a simple example to show that it is possible for taxes to cause increased welfare in a model of endogenous technological progress even when the fiscal revenue is thrown away. This is possible if inventors have a monopoly on their inventions and earn quasi rents which increase when labour supply increases. Imperfect competition implies that the market equilibrium is not first best and policy can have counterintuitive effects. In particular we have demonstrated that the wealth effect of a wasted lump-sum tax causes increased labour supply which implies an increased interest rate and faster growth. This can cause increased welfare for parameter values which many economists consider reasonable. We have started our enquiry by focusing on logarithmic preferences, because additive separability of the utility function offers greater tractability, but we are planning to extend our analysis to the case of a more general class of preferences. Also we will explore the effects of lump-sum taxes in a quality ladder model of growth driven by R&D.

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<sup>1</sup> $\chi$  is in fact the reciprocal of the elasticity of labour supply with respect to the wage, keeping constant the marginal utility of wealth, i.e. the so called Frisch elasticity.

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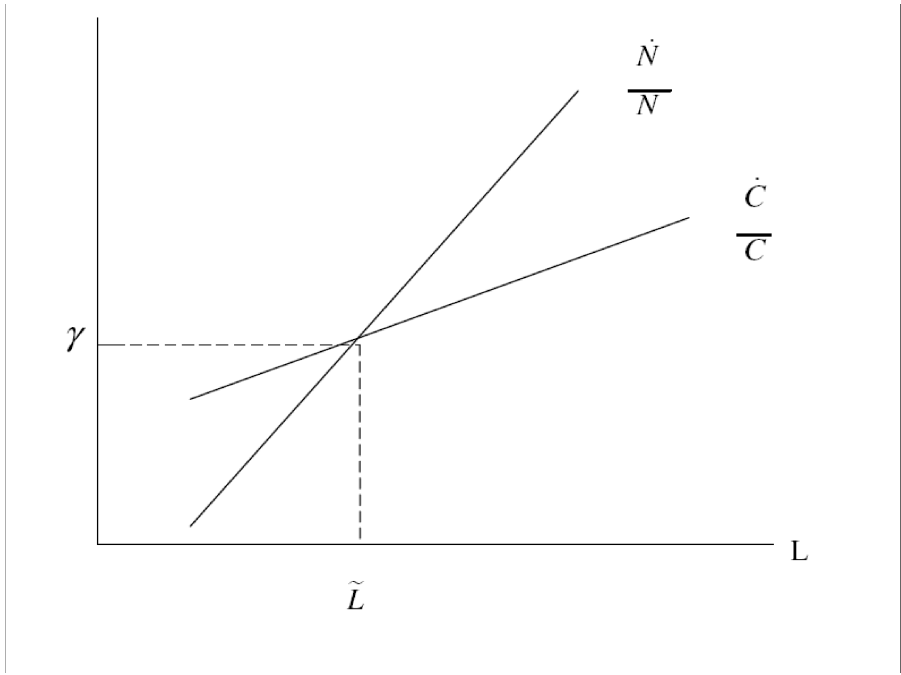


Figure 1:

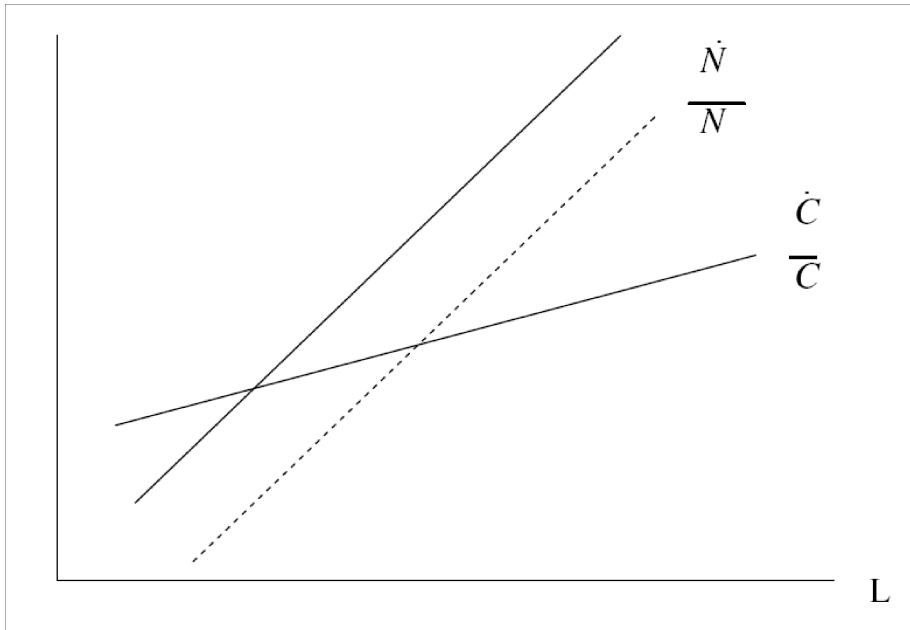


Figure 2: